Analytical Model for a 10 Cylinder Swash Plate Electric Compressor

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ABSTRACT

Recent advancements in the field of mobile air conditioning and refrigeration have witnessed an extensive use of the swash plate compressor due to its compact structure, continuous operation, small size, light weight and better thermal comfort inside the vehicle. The design of the swash plate compressor is complex so that it requires considerable contributions from different fields of engineering viz. engineering mechanics, heat transfer and fluid dynamics. An estimate of compressor performance through an analytical/mathematical model at the early stages of design and development serves as a useful tool for the designer. The input power, refrigerant mass flow rate, compression ratio and volumetric efficiency are important parameters to characterise the compressor performance. This paper presents an analytical/mathematical model for a 10-cylinder swash plate compressor with the emphasis on predicting its performance in terms of shaft torque and mass flow rate for a given rpm. A kinematic model is developed to obtain the piston displacement as an explicit function of angle of rotation of the swash plate. The model of piston and swash plate dynamics is developed then by analysing the interactions between forces and moments. The compression process model is formulated to determine the temperature and pressure inside the cylinder during one revolution of the swash plate along with the total mass flow rate in and out of the compressor. A time-varying model for the compressor is developed by combining the above three sub-models. Some experimental validation comparing predicted and measured drive torque has been done to verify the analytical/mathematical model. The predicted torque is in close agreement with the measured value.

Keywords: Air conditioning; swashplate; mathematical model

1. INTRODUCTION:

Swash-plate compressors are widely employed in mobile refrigeration and air conditioning applications due to their compact structure, light weight and small size (Srivastava et al. 2016). The design of such compressors is complex and requires an estimate of performance to be provided by the designer. Analytical/Mathematical models serve as useful tools during the early stages of design. The power and volumetric efficiency are important parameters for characterising the compressor performance. Estimation of shaft torque is important for design of the driving motor and sizing of the shaft and other mechanical components.

The existing literature on the modelling of swash-plate compressors can be broadly classified into two categories – mechanical design and thermo-mechanical performance. The first deals with the modelling of individual components of the compressor such as valve dynamics, friction in reciprocating parts and compressor mounting brackets. The second category includes models that aim towards predicting compressor performance such as

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kinematic and dynamic characteristics and evaluation of compressor performance including shaft torque and volumetric efficiency.

The dynamic analysis of a swash-plate was presented by Below et al. (1984) together with a mathematical model for stress analysis and bearing load calculation. An approximate sinusoidal motion for the piston was observed. Liu et al. (2012) investigated dynamic characteristics of the wobble-plate compressor using a geometric description of the wobble plate. The kinematic and dynamic characterstics of a six-cylinder variable-displacement wobble plate compressor were thoroughly analysed by Taguchi et al. (1990). An analytical model for the wobble plate compressor was developed by Tojo et al. (1990) and applied to a sample swash-plate compressor for some practical design guides. Tian et al. (2006) developed a steady-state mathematical/analytical model for a variable displacement swash-plate compressor by combining sub-models for piston and swash-plate dynamics, compression process and flow control valve. An analytical expression based on experimental data was presented for the compression model.

In the light of the literature it has been found that suction gas temperature, clearance volume and pressure losses across valves are the factors that may contribute in reducing volumetric efficiency of the compressor (Srivastava et al. 2016). Incorporating all the phenomenon responsible for volumetric inefficiencies in a model is a tedious job. Most of the existing literature focuses at modelling volumetric efficiency of hermetic compressors. An analytical formula for volumetric efficiency of hermetic compressors was expressed by Grolier et al. (2002) that includes the effect of suction gas temperature. Schreiner et al. (2010) presented simulation results on volumetric inefficiencies associated to the compression process of the compressor used for household refrigeration. A steady-state mathematical model for volumetric efficiency was presented by Darr et al. (1992). An analytical expression for isentropic efficiency based on experimental data was presented.

This paper presents an analytical/mathematical model for a 10-cylinder fixed-displacement swash-plate compressor with the emphasis on predicting its performance in terms of shaft torque. A kinematic model is developed to obtain the piston displacement as an explicit function of angle of rotation of the swashplate. The model of piston and swashplate dynamics is developed then by analysing the interactions between forces and moments. The compression process model is formulated to determine the temperature and pressure inside the cylinder during one revolution of the swashplate along with the total mass flow rate in and out of the compressor. A steady-operation model for the compressor is developed by combining the above three sub-models. Results are presented for a 10 cylinder fixed-displacement swash-plate compressor.

2. ANALYTICAL MODEL FOR INVESTIGATION OF SWASH PLATE ELECTRIC COMPRESSOR:

The present study considers a fixed-displacement swash-plate compressor with 10 cylinders. The pistons are mounted along the periphery of the swash-plate. The rotating motion of the swash-plate (a tilted disc mounted at a constant angle from the shaft axis) is converted to reciprocating motion of piston inside the cylinder. Cylinders are aligned on both sides of the swash-plate in such a way that compression in one cylinder causes suction in the symmetrically opposite cylinder. Pressure actuated reed valves are employed for both suction and discharge. **Fig. 1** gives an overview of the geometrical configuration of the compressor. Only one cylinder is shown for clarity. Z is the displacement of the top of the piston from its lowest point and s is the circumferential distance through which the swash-plate has rotated. ϕ is the angular location of the cylinder with respect to a datum line and θ is the angle through which the swash-plate has rotated. ω is the angular velocity (rad/s) and α is the angular acceleration (rad/s^2).

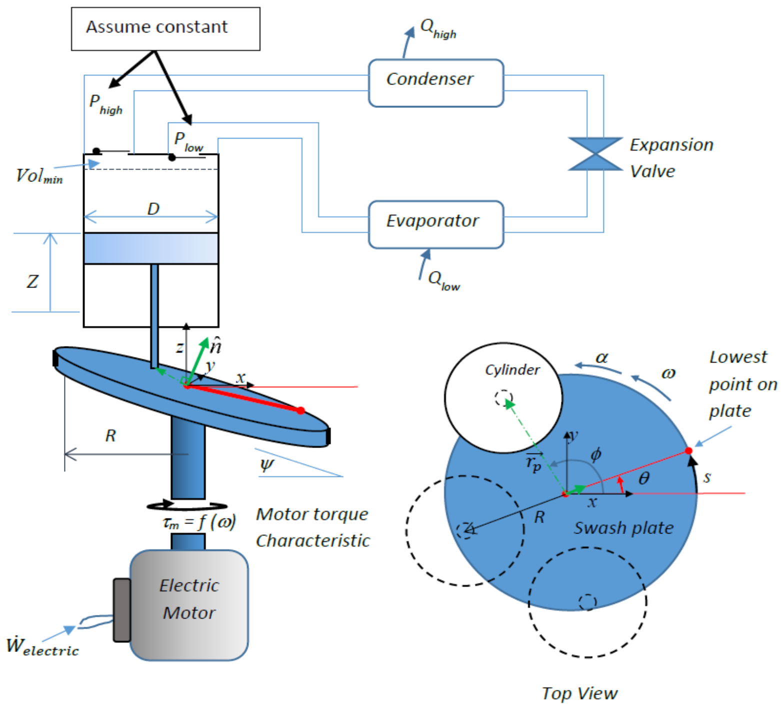


Figure 1 Swashplate electric compressor (only one cylinder shown for clarity)

2.1. Kinematics:

Assuming the top surface of the swash-plate is a single plane and $\hat{n}$ is a unit normal vector to that plane, then $\hat{n}$ is a function of θ, the angular position of the lowest point on the plate:

$$\hat{n} = \sin(\psi)\cos(\theta)\hat{i} + \sin(\psi)\sin(\theta)\hat{j} + \cos(\psi)\hat{k} \tag{1}$$

Consider a point, p on the plate in contact with the push-rod (connecting rod) for one of the pistons located at an angle ϕ shown in **Fig. 1**. The position vector of p for the coordinate system shown in **Fig. 1** will be:

$$\vec{r_p} = Rcos(\phi)\hat{\imath} + Rsin(\phi)\hat{\jmath} + z_p\hat{k} \tag{2}$$

Because $\vec{r_p}$ is in the plane for the plate, it will be at 90° to the normal and the dot product $\vec{r_p} \cdot \hat{n} = 0$. This gives:

$$z_p = -R\tan(\psi)(\cos\phi\cos\theta + \sin\phi\sin\theta) = -R\tan(\psi)\cos(\phi - \theta) \tag{3}$$

Therefore for the ith piston located at angle ϕ_i the displacement of the piston from bottom dead centre is given by:

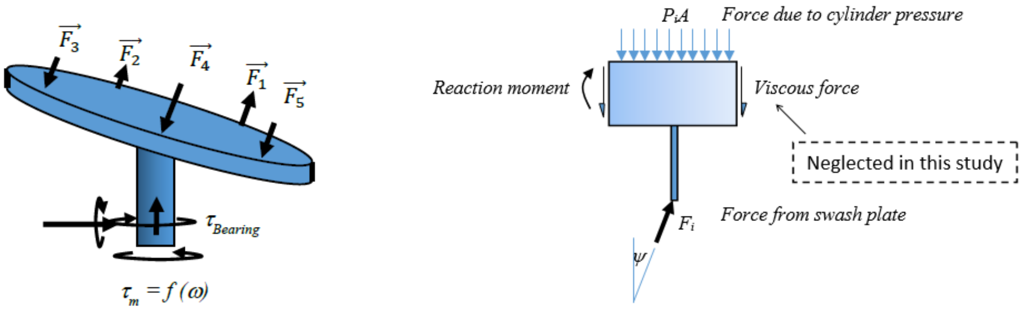
$$Z_i = R\tan(\psi)(1 - \cos(\phi_i - \theta)) \tag{4}$$

The velocity of the piston is related to the angular velocity by differentiating Eq. (4) with respect to time to get:

$$v_i = -R\tan(\psi)\omega\sin(\phi_i - \theta) \tag{5}$$

2.2. Dynamics:

Fig. 2 shows a free-body diagrams of the swash plate and piston. It is assumed that force from each cylinder acts in the direction perpendicular to the plate. There are also some reaction forces at the bearings.



(a) (b)

Figure 2 Free-body diagram (a) swash plate (b) piston

For rotation of the rigid body about the z-axis, Newton's 2nd Law gives:

$$\sum M_z = I_z\alpha \tag{6}$$

Where I_z is the rotational moment of inertia about the z-axis and α is the angular acceleration. To solve Eq. (6) the torque due to each of the reaction forces from the pistons shown in **Fig. 2** is needed. The moment vector of the force from the ith piston is given by the cross product:

$$\overrightarrow{M_i} = \overrightarrow{r_p} \times \overrightarrow{F_i} = F_i\overrightarrow{r_p} \times \hat{n} \tag{7}$$

Substituting Eq (1) and Eq. (2) into Eq. (7) and evaluating the component in the z-direction gives the torque about the axis exerted by the Force from the ith piston is:

$$\tau_i = F_i R\sin(\psi)\sin(\phi_i - \theta) \tag{8}$$

Substituting into Eq. (6) gives:

$$\tau_m - R\sin(\psi)\sum_{i=1}^{N} F_i\sin(\phi_i - \theta) - \tau_{bearing} = I_z\alpha \tag{9}$$

Where N is the total number of pistons.

Newton's 2nd Law applied in the z-direction for the piston gives:

$$F_i\cos(\psi) - P_i\frac{\pi D^2}{4} - \mu\frac{\pi DL}{\delta}v_i = ma_i \tag{10}$$

Where P_i is the pressure in the cylinder, D is the diameter of the cylinder, L is the length of the piston in contact with the wall of the cylinder, δ is the gap between the piston and the cylinder, μ is the dynamic viscosity of the lubricant, v_i is the velocity of the piston, m is the mass of the piston and a_i is the acceleration of the piston. Eqs (9) and (10) with (4) and (5) needed to be solved.

Neglecting the mass of the cylinder and viscous force in Eq. (10), the pressure in the cylinder is connected to the force applied by the swashplate using:

$$F_i = \frac{P_i\pi D^2}{4cos\psi} \tag{11}$$

For any angular position, if the mass of the swashplate and bearing resistance torque is neglected, the torque that needs to be supplied to the swashplate is:

$$\tau = Rsin(\psi)\sum_{i}^{N} F_i sin(\phi_i - \theta) \quad (12)$$

3. SWASHPLATE COMPRESSOR MODEL – IDEAL GAS WITH CONSTANT SPECIFIC HEAT:

This study considers the steady state case (constant rpm), ideal gas, isentropic compression, massless piston, and massless swashplate.

3.1. Mathematical Formulation:

To complete the model, pressures inside the cylinder as a function of the piston position and geometry are needed. The pressure in the cylinder P_i, needed in Eq. (10) is quite complicated. To a first approximation, the adiabatic processes are modelled using isentropic equations for an ideal gas with constant specific heats (Eq. (13)). A more accurate approach would be to use tables of data for the refrigerant.

$$P = P_0\left(Vol_0/Vol\right)^k \quad (13)$$

In Eq. (13) k is the ratio of specific heats for the refrigerant vapour. Based on some data from NIST chemistry webbook (Linstrom and W.G. Mallard. 2019), $k \approx 1.13$ might be a reasonable estimate for R134a vapour.

The simplest model to account for opening and closing of the valves is isentropic compression and expansion while the reed valves are closed and constant pressures P_{high} and P_{low} when the outlet and inlet valves are open respectively. For an ideal gas with constant specific heats this gives:

$$P_i = MIN\left(P_{high}, P_{low}\left(\frac{Vol_{max}}{Vol_i}\right)^k\right) \quad (14)$$

When the piston is going up and

$$P_i = MAX\left(P_{low}, P_{high}\left(\frac{Vol_{min}}{Vol_i}\right)^k\right) \quad (15)$$

When the piston is going down.

If it is assumed that the temperature of the vapour in the cylinder at the beginning of compression is the temperature of the superheated vapour in the suction line then the mass in the cylinder during compression while reed valves are closed can be found using the ideal gas equation:

$$m = \frac{P_{low}Vol_{max}}{RT_{suction}} \quad (16)$$

With this information, the temperature at any point during compression is given by the ideal gas equation:

$$T_i = \frac{P_i Vol_i}{mR} \quad (17)$$

Where R is the ideal gas constant:

$$R = \frac{R_{univ}}{M_{refrigerant}} \quad (18)$$

Where $M_{refrigerant}$ = 102.03 kg/kmol for R134a. When the reed valve is open it is simplest to assume that the temperature stays constant. When the reed valves are closed, the mass in the cylinder stays constant. From this, the mass flow rate depends on the velocity of the pistons and whether the valves are open or closed. For example, the contribution to the mass flow rate out when the outlet valve is open is given by:

$$\dot{m}_i = \rho v_i \pi \frac{D^2}{4} \tag{19}$$

Where the density ρ is calculated using the ideal gas equation with P_{high} and $T_{max.}$

3.2. Calculation Results:

The case of 200 cc compressor with 10 cylinders was considered. **Tables 1 & 2** give assumed geometry and conditions. Pressures were selected to correspond to those of an experimental test run.

Table 1. Assumed geometry

No. Cyl	Tot swept vol (cm³/ft³)	Clearance vol (cm³/ft³)	D (cm/ft)	R (cm/ft)
10	200/0.007	1/0.000035	3.1/0.10	3.5/0.11

Table 2. Assumed conditions

Psuction (Bar/kPa)	Pdischarge (Bar/kPa)	Tsuction (°C/°F)	Cp (kJ/kg.K)/ (Btu/lb·°F)	Cv (kJ/kg.K)/ (Btu/lb·°F)
3.8/380	21.32/2132	31.70/89.06	1.032/0.24	0.837/0.19

Fig. 3 (a) shows the instantaneous total mass flow rate in and out of the compressor during one revolution. The pulses from the individual cylinders are clearly evident. **Fig. 3 (b)** shows the pressure and temperature inside one of the cylinders during a complete revolution of the swash-plate.

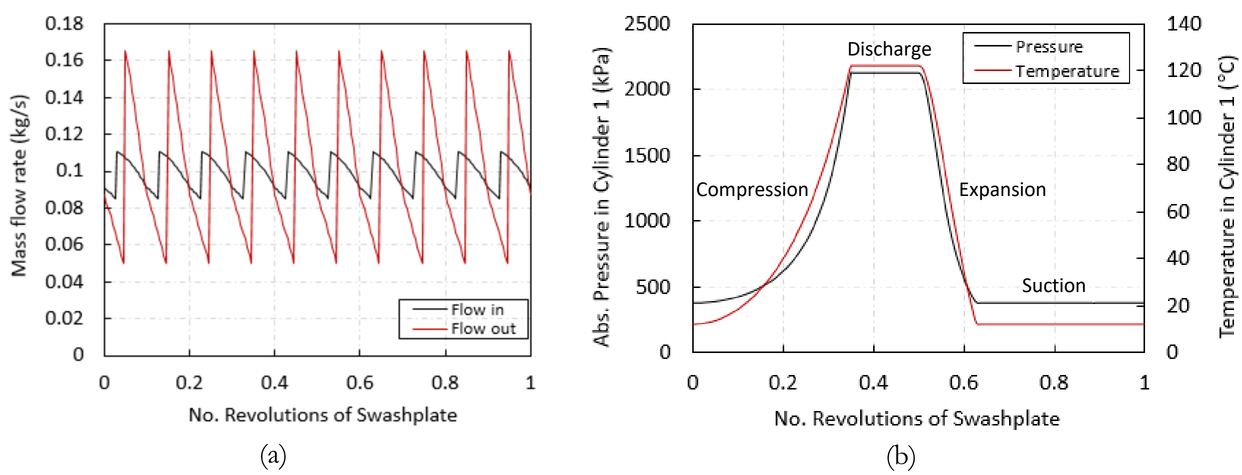


Figure 3 (a) Mass flow rate of refrigerant during one revolution of swashplate (b) Pressure and temperature inside one cylinder during one revolution of swash-plate

4. EXPERIMENTAL VALIDATION:

The experiment was performed at SuperCool Asia Pacific testing facilities. The setup consists of a variable capacity swash-plate compressor (200 cc/rev), a condenser (capacity performance of 7.6 kW), a universal refrigerator (cooling capacity of 9.4 kW), and a thermal expansion valve (TXV). It also contains a drier and an oil separator as auxiliary equipment.

A 30 kW induction motor controlled by a variable frequency drive (frequency inverter) was connected to the compressor to control the rotational speed to investigate a range of speeds at which the compressor works in an actual vehicle, and provide the related data such as torque (Nm), power (kW) and the speed (RPM) of the compressor. **Fig. 4** shows the schematic diagram of the experimental setup. 15kW and 10kW electric heaters were placed in the condenser and evaporator rooms to supply heated air, and a heater exchanger unit to remove hot air was also located in the condenser room. T-type air thermocouples and K-type probes are mounted on the system to obtain the refrigerant temperatures and pressures. The sensors are connected to two data loggers to send the information to a computer. Investigation of the collected data was carried out numerically in a spreadsheet.

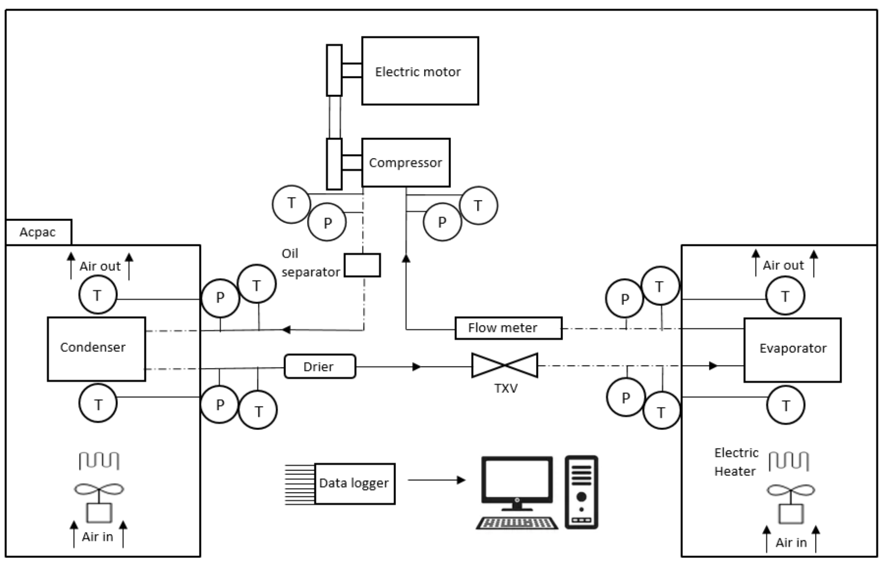


Figure 4 Schematic diagram of the experimental setup

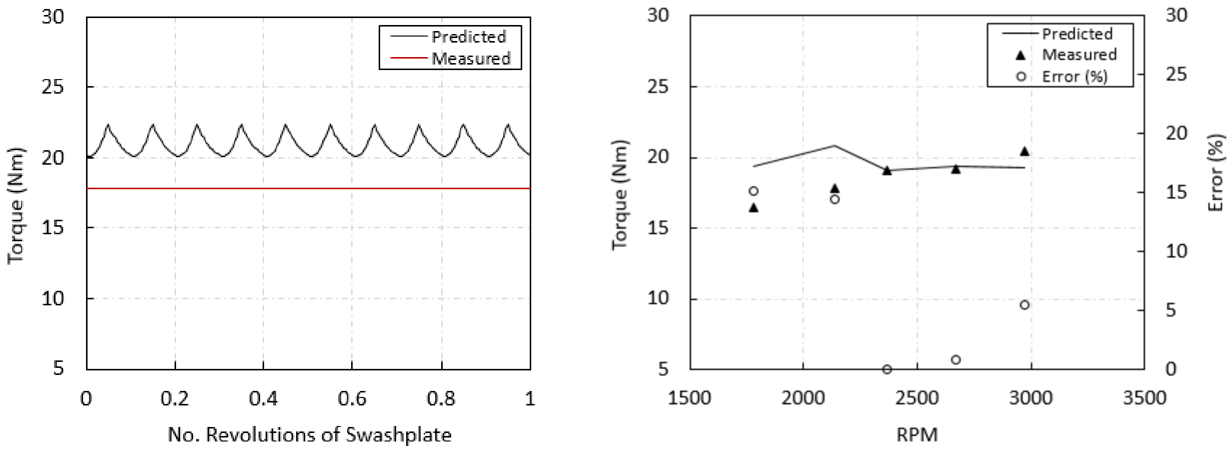
The measured average torque at different revolutions per minute (rpm) is outlined below in **Table 3**. The conditions correspond to the initial stages of operation – not long after starting up. Hence the suction temperatures are still high. These conditions are of interest for design since the torque is higher than it is for steady operation of the air conditioner.

Table 3. Average torque (22 ºC/71.6 ºF ambient)

RPM	Suction P (Bar/kPa)	Suction T (°C/°F)	Discharge P (Bar/kPa)	Discharge T (°C/°F)	Average Torque (N.m/ft.lb)
1778	4.10/410	29.5/85.1	20.00/2000	49.85/121.71	16.42/12.11
2135	3.80/380	31.70/89.06	21.32/2132	56.05/132.89	17.85/13.16
2370	3.80/380	30.75/87.35	17.99/1799	41.95/107.51	19.08/14.07
2670	3.70/370	30.90/87.62	18.89/1889	46.00/114.8	19.22/14.17
2974	3.60/360	30.75/87.35	19.20/1920	45.10/113.18	20.42/15.06

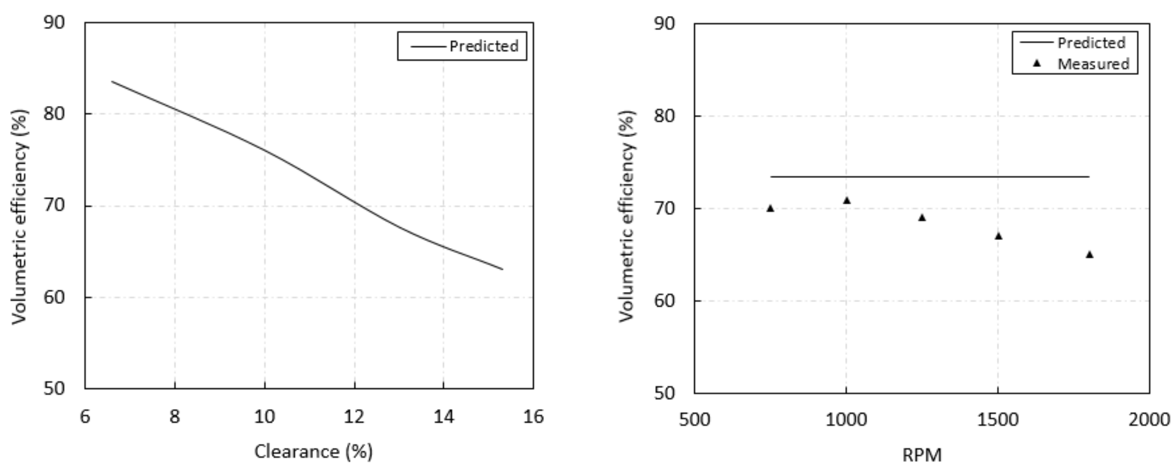
The validation is done for a 10 cylinder fixed displacement swash-plate compressor at 2135 rpm as depicted in **Fig. 5(a)**. The predicted torque is certainly in the range of the experimental data shown in **Fig. 5**. The trend with rpm shown in **Fig. 5(b)** is different for the experiment compared with the simulation. The experimental data show a steady increase with increasing rpm while the simulation is almost independent of rpm except for the case at 2135 rpm. The higher value can be attributed to the higher discharge pressure shown in **Table 3** for 2135 rpm, which is used as a model input.

From the results shown in **Fig. 5**, it is reasonable to conclude that the main contribution to the torque can be explained by the compression of the gas. It is conceivable that the extra resistance from the friction of the piston could bring the predictions closer to the experimental values concerning the trend of increasing torque with rpm. The isentropic efficiency is 100% as the compression process assumed to be isentropic. The variation of volumetric efficiency with clearance values and rpm is shown in **Fig. 6**. The model overpredicts volumetric efficiency. The temperature during compression is high **(Fig. 3(b))**. Non-ideal gas properties should help. Including the masses/inertia of the pistons and swash plate will make the model more complete for future studies.



(a) (b)

Figure 5 Experimental validation of simulated torque (a) at 2135 rpm (b) at different rpm



(a) (b)

Figure 6 Variation of volumetric efficiency with (a) clearance values (b) rpm at 11% of clearance

5. CONCLUSION:

This paper presents an analytical/mathematical model for a 10-cylinder swash plate compressor with the emphasis on predicting its performance in terms of shaft torque. A kinematic model is developed to obtain the piston displacement as an explicit function of angle of rotation of the swashplate. The model of piston and swashplate dynamics is developed then by analysing the interactions between forces and moments. The compression process model is formulated to determine the temperature and pressure inside the cylinder during one revolution of the swashplate along with the total mass flow rate in and out of the compressor. Some experimental validation comparing predicted and measured drive torque has been done to verify the analytical/mathematical model. The predicted and measured torque are in reasonable agreement with each other. The shaft torque can further be used for design and selection of the shaft at early stages of the design. It is reasonable to conclude that the main contribution to the torque can be explained by the compression of the gas. It is conceivable that the extra resistance from the friction of the piston can bring predictions closer to the experimental values. Including the masses/inertia of the pistons and swash plate will make the model more complete.

NOMENCLATURE

Z	-	Displacement of the piston, m
ϕ	-	Agular Location of the Cylinder
θ	-	Swashplate rotation angle, radian
ω	-	Angular Velocity, rad/s
α	-	Angular Acceleration, rad/s^2
τ_m	-	Motor Torque, N.m
μ	-	Dynamic Viscosity, Pa.s
P_{low}	-	Low Pressure, (Pa, kPa)
P_{high}	-	High Pressure, (Pa, kPa)

REFERENCES

Srivastava, P., Sharma, K., and Jha, R., "Modelling of Fixed Displacement Swash Plate Compressor: A Mathematical Approach towards Calculation of Shaft Torque and Volumetric Efficiency," SAE Technical Paper 2016-28-0172, 2016, doi: 10.4271/2016-28-0172.

Below, J.F. and Miloslavich, D.A., “Dynamics of swash plate mechanism,” International Compressor Engineering Conference, 1984.

Liu, Y.G. and Hui, G., “Motion simulation and dynamic analysis of a four cylinder wobble plate compressor,” Proceedings of international conference on mechanical engineering and material sciences (MEMS 2012), Paper 437 (2012)

N. Ishii, Y. Abe, T. Taguchi, et al., Dynamic behavior of variable displacement wobble plate compressor for automotive air conditioners, Proceedings of the 1990 International Compressor Engineering Conference at Purdue, USA (1990) 345-53

K. Tojo, K. Takao, M. Ito, et al., Dynamic behavior of variable displacement compressor for automotive air conditioners, SAE Trans 99 (6) (1990) 40-4

C.Q. Tian, Y.F. Liao, X.T. Li, A mathematical model of variable displacement swash plate compressor for automotive air conditioning system, International Journal of Refrigeration 29 (2006) 270-280

Grolier, P., “A Method To Estimate The Performance Of Reciprocating Compressors” International Compressor Engineering Conference, 2002, Paper 1510.

Schreiner, Joao Ernesto; Deschamps, Cesar J.; and Barbosa, Jader R., "Theoretical Analysis of the Volumetric Efficiency Reduction in Reciprocating Compressors due to In-Cylinder Thermodynamics" International Compressor Engineering Conference, 2010.

Darr, J. H. and Crawford, R. R., "Modelling of an Automotive Air Conditioning Compressor Based on Experimental Data," ACRC Technical Report, University of lllinois at Urbana-Champaign, 1992.

NIST Chemistry WebBook, NIST Standard Reference Database Number 69, Eds. P.J. Linstrom and W.G. Mallard, National Institute of Standards and Technology, Gaithersburg MD, 20899, https://doi.org/10.18434/T4D303, (retrieved June 27, 2019).